Proceedings of International Conference on Violent Flows (VF-2007)
Organized by RIAM, Kyushu University, Fukuoka, Japan

A B-spline Galerkin Scheme for Computing Hydroelastic Behavior of a Very Large Floating Structure

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Abstract

A new calculation scheme is firstly described for the pressure distribution method, giving directly the pressure on the bottom of a very large floating structure with shallow draft. The scheme utilizes bi-cubic B-spline functions for representing the unknown pressure and a Galerkin method for converting the integral equation into algebraic simultaneous equations. Careful consideration is paid to reduce the computation time with keeping numerical accuracy. Excellent performance of the scheme is confirmed by checking the Haskind relation and energy-conservation principle for all computed wavelengths. Secondly, a mode-expansion method is presented for solving the vertical vibration equation of a rectangular plate. Convergence of elastic responses with increasing the number of specified mode shapes is confirmed for head and oblique waves of very short wavelengths.

Keywords: Hydroelasticity; Very large floating body; Galerkin scheme; Hydrodynamic forces; Pressure distribution

1 Introduction

There is a strong need for very large floating structures which are to be used for airports, storage or manufacturing facilities, habitation, and so on. This need comes from the lack of adequate land space near metropolitan areas and/or environmental concerns such as pollution and noise associated with having an airport near residential areas.

Recently in Japan, as a national project, a floating airport is considered and its safety and performance in waves are studied. The configuration primarily considered is of pontoon type because of possibly easier construction and maintenance, and its size is of order of 5 km long and 1 km wide. Since the draft will be very small compared to dimensions of the plan view, this type of structure will be very flexible and elastic deformations may be more important than the rigid-body motions.

To analyze these elastic deformations, several analytical techniques have been proposed. Among them, probably the most common one is the mode-expansion method, representing the structural deflection by a superposition of a set of ‘general’ mode shapes. One of the difficulties in this method is that the first-order hydrodynamic forces corresponding to each of specified mode shapes must be accurately calculated even in the region of very short wavelengths.

The structure under consideration can be approximated by a zero-draft rectangular plate and thus hydrodynamically by the pressure distribution on the free surface. This approximation is known as the pressure distribution method, which was studied first by Yamashita [1]. Several authors, e.g. Ikoma *et al*. [2] and Yago [3], showed numerical results based on this method. However their accuracy in short wavelength region seems not enough, because the zero-th order approximation is adopted in the discretization of the integral equation for unknown pressures. Besides, with zero-th order panel method, the number of unknowns must be $O(10^4)$ for wavelengths of interest, and thus the computation time will be far from practical.

As a premise for structural analyses, we are required to develop a method which can compute hydrodynamic forces very accurately with fewer unknowns and less computation time. In the first half of the present paper, a new method is described of solving the integral equation in the pressure distribution method. To reduce the number of unknowns, bi-cubic B-spline functions are employed for representing unknown pressures.

2 Mathematical Formulation

Cartesian coordinates are defined, as in Fig.1, with $z = 0$ as the plane of undisturbed free surface and $z = h$ as the horizontal bottom. The plan view of the structure is rectangle with length L and width B, and the draft is assumed zero due to very small value relative to L and B.

Time-harmonic motions of small amplitude are considered, with the complex time dependence $e^{i\omega t}$ applied to all first-order oscillatory quantities. The boundary conditions on the body and free surface are linearized, and the potential flow is assumed. The incident angle of incoming wave is denoted by β.

Then we write the velocity potential ϕ, pressure p, and vertical displacement of the structure w, in the following normalized form:

$$\phi = i\omega a\{\phi_I + \phi_S\} + \sum_j i\omega X_j \phi_j \quad (1)$$

$$p = \rho g \left[a\{p_I + p_S\} + \sum_j X_j p_j \right] \quad (2)$$

$$w = a\{\zeta_I + \zeta_S\} + \sum_j X_j \zeta_j \quad (3)$$

where a is the amplitude of incident wave, ρ the fluid density, and g the gravitational acceleration.

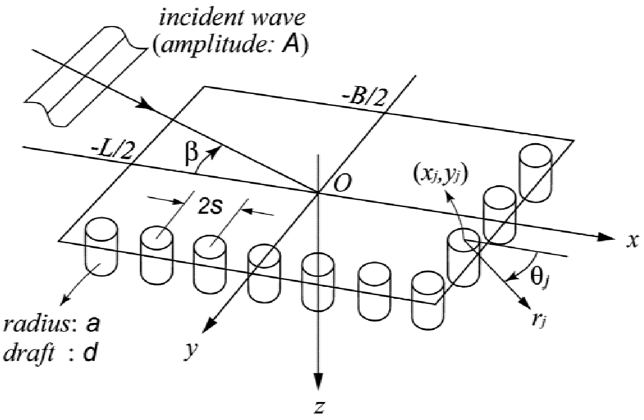

Fig.1 Coordinate system and notations

Suffix I represents quantities related to the incident wave, S the scattering component, and j the radiation component of j-th mode of motion with complex amplitude X_j. In the definition of mode indices, not only rigid-body motions, but also a set of 'general' modes to be used for representing elastic deformations are included.

In the analyses to follow, the length dimensions are nondimensionalized in terms of $L/2$, and thus the structure exists in the region of $|x| \le 1$ and $|y| \le b \equiv B/L$ on z = 0.

When the pressure is applied on the free surface, the dynamic and kinematic free-surface boundary conditions are given as

$$p_j = K\phi_j + \zeta_j, \quad \frac{\partial \phi_j}{\partial z} = \zeta_j \quad \text{on } z = 0 \quad (4)$$

where $K=\omega^2/g$, and $p_j = 0$ outside of the structure.

3 Results and Discussion

3.1 Numerical accuracy and computation time

Computations were performed for a rectangular plate of $L/B = 5.0$. Since our interest is placed on short wavelength region, the finite-depth effect may be negligible and thus the water depth was taken as infinity.

The discretization of the plate into panels is made such that the first quadrant (x > 0 and y > 0) is subdivided into NX in the x-axis and NY in the y-axis with ratio NX/NY = 5.0, meaning that each panel is a square.

The number of mode functions defined in Table 1 was taken equal to 25 in checking the numerical accuracy, and all computations were implemented, using an engineering workstation of HP 9000 series / model 735.

Table 1 Dimensions of a body and other parameters for numerical computations

Length ($a = L/2$)	2.0
Breadth ($b = B/2$)	2.0
Draft (d)	0.5
Water depth (h)	2.0
Separation distance (s/a)	0.01, 0.05, 0.1036, 0.2071
Wave frequency (Ka/π)	0.5 ~ 5.0
Incident-wave angle (β)	0 deg.

It can be clearly seen that the results approach smoothly converged values as the number of panels increases. Rough estimate from Fig.3 indicates that reliable results may be obtained, provided $L/\lambda < 0.8*NX$ is satisfied. Considering that the computed results are very smooth at least for computed points of L/λ, there seem to be no phenomena of irregular frequencies. (This is also the case for the added mass, though the results are not shown here.)

4 Conclusion

In the first half of this paper, a new calculation scheme was described for the pressure distribution method. The scheme utilizes bi-cubic B-spline functions for representing the unknown pressures, and employs a Galerkin method for converting the integral equation into a linear system of simultaneous equations. The developed scheme was confirmed to be effective, particularly for shorter wavelengths up to the order of $L/\lambda = 50$, in terms of good accuracy with fewer unknowns and relatively less computation time.

In the second half of the paper, the vertical vibration equation of a rectangular flat plate was solved, with free-end boundary conditions satisfied in the process of transforming the stiffness matrix by partial integrations.

References

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